Schwartz
11.6 (a)

We wish to show $\bar{\psi}_L \gamma_\mu \psi_R = \bar{\psi}_R \gamma_\mu \psi_L = 0$

We are working with Left and Right handed spinors, so we are implicitly working in the Dirac basis, Weyl Rep.

Then $\bar{\psi}_L \gamma_\mu \psi_R = \overline{\left[\frac{1}{2}(1-\gamma^5)\psi\right]} \gamma_\mu \left[\frac{1}{2}(1+\gamma^5)\psi\right]$

$$= \frac{1}{4}\left[(1-\gamma^5)\psi\right]^\dagger \gamma_0 \gamma_\mu (1+\gamma^5)\psi$$

$$= \frac{1}{4}\psi^\dagger \underbrace{(1-\gamma^5)^T \gamma_0 \gamma_\mu (1+\gamma^5)}\psi$$

$$(1-\gamma^5)^T \gamma_0 \gamma_\mu (1+\gamma^5)$$

$$= \begin{pmatrix}1 & \\ & 0\end{pmatrix}\begin{pmatrix} & 1\\ 1 & \end{pmatrix}\begin{pmatrix} & \sigma\\ \bar{\sigma} & \end{pmatrix}\begin{pmatrix}0 & \\ & 1\end{pmatrix}$$

$$= \begin{pmatrix} & 1\\ 0 & \end{pmatrix}\begin{pmatrix} & \sigma\\ 0 & \end{pmatrix} = \boxed{0}$$

Similarly, $\bar{\psi}_R \gamma_\mu \psi_L = \frac{1}{4}\psi^\dagger \underbrace{(1+\gamma^5)^T \gamma_0 \gamma_\mu (1-\gamma^5)}\psi$

$$(1+\gamma^5)^T \gamma_0 \gamma_\mu (1-\gamma^5) = \begin{pmatrix}0 & \\ & 1\end{pmatrix}\begin{pmatrix} & 1\\ 1 & \end{pmatrix}\begin{pmatrix} & \sigma\\ \bar{\sigma} & \end{pmatrix}\begin{pmatrix}1 & \\ & 0\end{pmatrix}$$

$$= \begin{pmatrix} & 0\\ 1 & \end{pmatrix}\begin{pmatrix} & 0\\ \bar{\sigma} & \end{pmatrix}$$

$$= \boxed{0}$$

(b) In rest frame, we can write

$$u_\uparrow = \cancel{\ } \begin{pmatrix} \xi_\uparrow \\ \xi_\uparrow \end{pmatrix}, \quad u_\downarrow = \begin{pmatrix} \xi_\downarrow \\ \xi_\downarrow \end{pmatrix},$$

where $\xi_\uparrow \xi_\downarrow = 0$,

$\xi_\uparrow, \xi_\downarrow$ are 2D ~~vector~~ objects

$$\bar{u}_\uparrow \gamma_\mu u_\downarrow = (\xi_\uparrow^* \ \xi_\uparrow^*) \gamma_0 \gamma_\mu \begin{pmatrix} \xi_\downarrow \\ \xi_\downarrow \end{pmatrix}$$

$$= (\xi_\uparrow^* \ \xi_\uparrow^*) \begin{pmatrix} & \mathbb{1} \\ \mathbb{1} & \end{pmatrix} \begin{pmatrix} & \sigma_\mu \\ \bar{\sigma}_\mu & \end{pmatrix} \begin{pmatrix} \xi_\downarrow \\ \xi_\downarrow \end{pmatrix}$$

$$= \xi_\uparrow^* \bar{\sigma}_\mu \xi_\downarrow + \xi_\uparrow^* \sigma_\mu \xi_\downarrow$$

Recall $\sigma_\mu = (\mathbb{1}, \vec{\sigma})$, $\bar{\sigma}_\mu = (\mathbb{1}, -\vec{\sigma})$, so all 3-space components vanish, but by orthogonality of $\xi_\uparrow, \xi_\downarrow$, the 0-th component vanishes as well.

Now for v, In rest frame,

$$v_\uparrow = \begin{pmatrix} \eta_\uparrow \\ -\eta_\uparrow \end{pmatrix}, \quad v_\downarrow = \begin{pmatrix} \eta_\downarrow \\ -\eta_\downarrow \end{pmatrix}, \quad \eta_\uparrow, \eta_\downarrow \text{ 2D objects, } \cancel{\text{2D vector}}$$

$$\eta_\uparrow \eta_\downarrow = 0$$

$$\bar{v}_\uparrow \gamma_\mu v_\downarrow = \begin{pmatrix} \eta_\uparrow^* & -\eta_\uparrow^* \end{pmatrix} \begin{pmatrix} & \mathbb{1} \\ \mathbb{1} & \end{pmatrix} \begin{pmatrix} & \sigma_\mu \\ \bar{\sigma}_\mu & \end{pmatrix} \begin{pmatrix} \eta_\downarrow \\ -\eta_\downarrow \end{pmatrix}$$

$$= \begin{pmatrix} -\eta_\uparrow^* & \eta_\uparrow^* \end{pmatrix} \begin{pmatrix} -\sigma_\mu \eta_\downarrow \\ \bar{\sigma}_\mu \eta_\downarrow \end{pmatrix}$$

$$= \eta_\uparrow^* \sigma_\mu \eta_\downarrow + \eta_\uparrow^* \bar{\sigma}_\mu \eta_\downarrow$$

By same reasoning as for u, this vanishes.

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